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The invention of clocks with minute hands as well as hour hands gave the next variant, as to when both hands would be together—a relatively modern form of the question, as is also the astronomical problem of the occurrence of the new moon. The latest form, however, has to do with the practical question of a railway timetable, but here graphic methods naturally take the place of analysis so that of all the variants those of the couriers and the clock hands seem to be the only ones that will survive. Neither is valuable *per se,* but each is interesting, each is real within the range of easy imagination, and each involves a valuable mathematical principle—a fairly refined idea of function, and so it is probable that each will persist in spite of the present transitory period of the attempted debasement of elementary mathematics.

AN INVERSION OF THE COMPLETE QUADRILATERAL.

By J. W. CLAWSON, Ursinus College.

It is the purpose of this paper to point out an interesting example of the method of inversion. If a complete quadrilateral with some of its related lines and circles be inverted with respect to the quadrilateral's Wallace point, a new complete quadrilateral with some of its related circles and lines results. It is remarkable that this new quadrilateral is inversely similar to the original one, as will appear from II below.

Four straight lines (Fig. 1) ARB, BCP, CQA, PQR form a complete quadrilateral with A, P; B, Q; C, R for opposite vertices. The lines taken three by three also determine four triangles ABC, AQR, BRP, CPQ. It is well known that the circles circumscribing* these triangles are concurrent at a point O, the *Wallace point* of the quadrilateral,[1] that the circumcenters of the triangles are concyclic on l, the *circumcentric circle,*[2] that the orthocenters of the triangles are

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tion, folio 64); Albert (1540, 1561 edition, folio Pi); Baker (1568, 1580 edition, folio 36); Coutereel (1631 edition, p. 371, and Eversdyck edition of 1658, p. 403); Trenchant (1566, 1578 edition, p. 280); Wentsell (1599, p. 51); Peletier (1549, 1607 edition, p. 290), Vander Schuere (1600, folio 179); Schonerus (notes on Ramus, 1586 edition, p. 174), and many others. Köbel's rather quaint German is interesting: "Zwen Burger vsz Oppenheym / einer Sō Heynrich / der ander Contz vō Treber gnant / wolten mit einander gen Rom gehn / vn̄ Heinrich was alt / vn̄ mocht einn tag nit mehr dan̄ zehen meiln gehn / Aber Contz vō Treber was jung vnnd starck / der mocht einen tag 13. meilen gehn / Deszhalben gieng Son Heynrich neun tag eh ausz Oppenheym dann Contz von Treber / Also war Son Heynrich Contzen 90. meilen furgangen / eh Contz angehaben hat auszugehn.

"Nun ist die frag / inn wie vil tagen Contz von Treber / Son Heynrichen übergangen / vnnd die zwen zusamen kommen seind." See also his *Zwey rechenbüchlin,* Frankfort, 1537 edition, folio 84.

* This figure is slightly distorted. The circles should join exactly through the points R and Q.

[1] "Scoticus," Leybourn's *Mathematical Repository,* 1804, Vol. I, p. 170. Mackay, *Proc. Edin. Math. Soc.,* Vol. IX.

[2] Davies, *Math. Repos.,* 1835, Vol. VI, Question 555 answered.

[1, 2] Steiner, Gergonne's *Annales,* 1828, Vol. XVIII, pp. 302, 303, 1°, 2°, 3°, 4°.

collinear on o, the *orthocentric line*,[3] and that the feet of the perpendiculars from O on the four lines are collinear on p, the *pedal* line[4] of the quadrilateral.

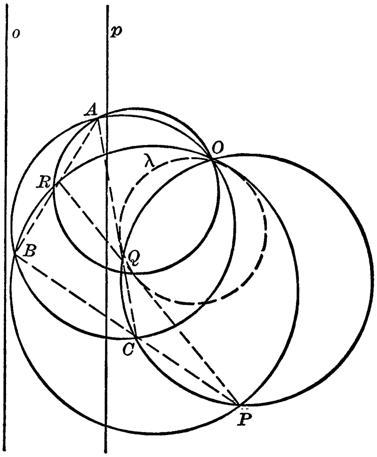

FIG. 1.

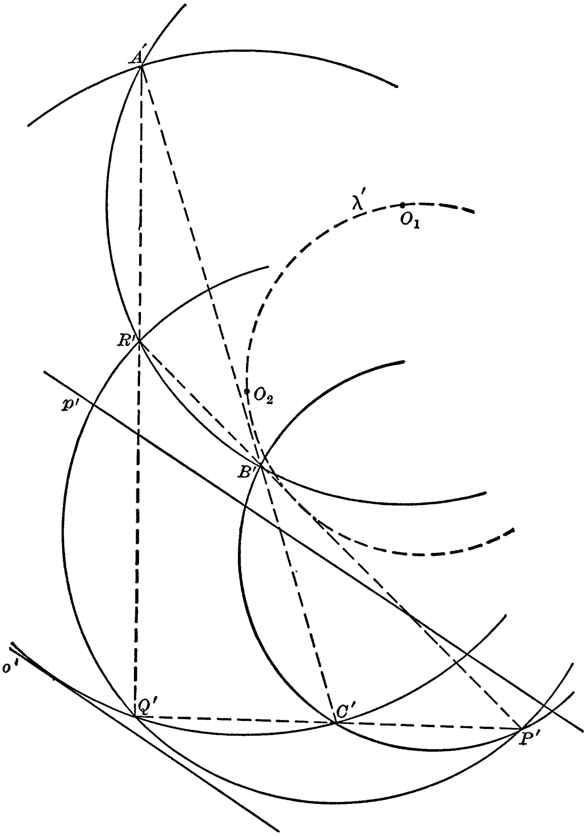

FIG. 2.

Invert the figure with O as center of inversion and with any radius. The full lines of Fig. 1 invert into broken lines of Fig. 2 and vice versa.

I. The circles ABC, AQR, BRP, CPQ of Fig. 1 invert into four straight lines of Fig. 2, $A'B'C'$, $A'R'Q'$, $P'B'R'$, $P'Q'C'$, which form a new quadrilateral with P', A'; Q', B'; R', C' for opposite vertices. The straight lines ARB, BCP, AQC, PQR invert into the four circles, concurrent at O' (not shown on this figure), which circumscribe the triangles $A'R'B'$, $B'C'P'$, $C'A'Q'$, $P'Q'R'$.[5]

II. The new quadrilateral is inversely similar to the old, the center and axis of similitude being respectively O and the bisector of the angle AOP. (This line also bisects angles BOQ and COR.)

For angle A is equal to (or supplementary to) the angle between the lines joining A' to the centers of the circles $A'R'B'$ and $A'C'Q'$. Denote these centers

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[1, 3, 4] CASEY, *Sequel to Euclid*, ed. of 1886, pp. 35, 36.

[5] See MCCLELLAND, *Geometry of the Circle*, p. 256, Ex. 12.

by O_1 and O_2. Then $\angle O_1A'O_2 = \angle O_1O'O_2 = \angle O_1O'A' + \angle O_2O'A' = \angle O'B'A' - \angle O'C'A' = \angle C'O'B' = \angle P'$. Thus $\angle A = \angle P'$. Similarly any angle in the figure $APBQCRO$ can be proved equal to the corresponding angle in the figure $P'A'Q'B'R'C'O'$.

III. The orthocentric line of either quadrilateral inverts into the circumcentric circle of the other.

For the pedal line, p, passes through the intersection of BCP and a line through O perpendicular to that line. Hence its inverse is a circle passing through the intersection of the circle $O'B'C'P'$ and a straight line through O' orthogonal to that circle. Hence the inverse of p is a circle passing through the opposite extremities of the diameters through O' of the circles $A'R'B'$, $B'C'P'$, $C'A'Q'$, $P'Q'R'$. Hence the inverse of the orthocentric line, o, which is parallel to p and twice as far from O,[6] is a circle touching the inverse of p at O' and with a radius half as large; that is, it is the circumcentric circle.[7]

IV. Many properties of a complete quadrilateral lead, on inversion, to new properties of this figure. However the results are often complicated and of little interest. One example will be given.

The theorem: "The circles on the three diagonals of the quadrilateral as diameters are coaxial, the radical axis being the orthocentric line,"[8] inverts into:

"The three circles, (1) through A, P orthogonal to the circle OAP, (2) through B, Q orthogonal to OBQ, (3) through C, R orthogonal to OCR, are coaxial; and the circumcentric circle belongs to the same coaxial system."

A PROBLEM IN PROBABILITY.[1]

By C. S. JACKSON, R. M. Academy, Woolwich, England.

1. A problem first proposed by De Moivre and extended by Simpson was thrown into the following form by Laplace: If the numerical result of a single trial is equally likely to have any value between 0 and b, the chance that after n trials the sum of the results obtained shall be less than a is

$$\text{(i)} \qquad \frac{1}{b^n n!}\{a^n - n_1(a-b)^n + n_2(a-2b)^n \cdots\},$$

n_r denoting $n!/[r!(n-r)!]$ and the series being continued as long as $a - rb$ is positive.[2] In the following note an alternative mode of investigating (i) is used, which is intended to illustrate how each term of the formula arises.

2. Let $x_1 \cdots x_n$ be n positive items, each equally likely to have any value

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[6] Steiner, *loc. cit.* Davies, *loc. cit.* Casey, *loc. cit.*

[7] Cf. McClelland, *loc. cit.*

[8] Durell, *Plane Geometry for Advanced Students*, Part I, Theorem 86, p. 188.

[1] The proof sheets of this article never reached us from the author, having probably been lost in ocean transit. Editors.

[2] See Todhunter, *History, etc., of Probability*, pp. 84, 208, 542.